

EPR systém

$$\mathcal{H} = \mathcal{H}_e \otimes \mathcal{H}_p = \mathcal{H}_e \oplus \mathcal{H}_p$$

$$\hat{S}[\vec{\alpha}] = \hat{S}_L[\vec{\alpha}] + \hat{S}_P[\vec{\alpha}] \quad \hat{S} = \frac{\hbar}{2} (\hat{G}_L + \hat{G}_P)$$

$$\textcircled{1} \quad \hat{S}^2 = \hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2 = ?$$

$$\textcircled{2} \quad \text{maléžme } \hat{P}_0, \hat{P}_1 \rightarrow \text{aležme } \hat{P}_0, \hat{P}_1 \text{ třídy } l(l+1) \\ \hat{S}^2 = \hbar^2 (0 \hat{P}_0 + 1 \hat{P}_1) \quad \hat{P}_0 + \hat{P}_1 = \hat{I}$$

$$\hat{P}_{EPR} = \frac{1}{4} (\hat{I} - \hat{G}_L \cdot \hat{G}_P)$$

$$|EPR\rangle = \frac{1}{\sqrt{2}} (|\ell\vec{e}\rangle |\nu\vec{e}\rangle - |\ell\vec{e}\rangle |\nu\vec{e}\rangle)$$

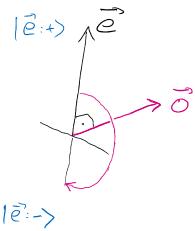
$$|\vec{e}|=1 \quad |\vec{e}\rangle = \hat{G}[\vec{e}] |\vec{e}\rangle \quad \vec{e} \cdot \vec{o} = 0$$

řešení:

$$\textcircled{1} \quad \hat{S}^2 = \hat{S} \cdot \hat{S} = \frac{\hbar^2}{2} (3\hat{I} + \hat{G}_L \cdot \hat{G}_P) \quad \hat{P}_1^2 = \hat{P}_1$$

$$\textcircled{2} \quad \hat{P}_1 = \frac{1}{4} (3\hat{I} + \hat{G}_L \cdot \hat{G}_P) \quad \hat{P}_0^2 = \hat{I} - \hat{P}_1$$

$$\hat{P}_0 = \frac{1}{4} (\hat{I} - \hat{G}_L \cdot \hat{G}_P) \quad \text{Tr } \hat{P}_0 = 1 = \dim \mathcal{H}_0 \quad \text{Tr } \hat{P}_1 = 3 = \dim \mathcal{H}_1$$



$$\textcircled{4} \quad \hat{P}_{EPR} |EPR\rangle = |EPR\rangle \quad \Leftrightarrow \quad \hat{P}_{EPR} = \underbrace{|EPR\rangle \langle EPR|}_{\text{ověřte pravou dosazenou}} \quad \text{na } \hat{P}_{EPR} \text{ a } |EPR\rangle \text{ (UF!!!)}$$

$$\textcircled{3} \quad \hat{S}[\vec{\alpha}] |EPR\rangle = 0 \quad \Rightarrow \quad \hat{S}^2 |EPR\rangle = 0 \quad \Rightarrow \quad \hat{P}_{EPR} |EPR\rangle = |EPR\rangle$$

$$1) \quad \vec{\alpha} = \alpha_x \vec{e} \\ \hat{S}[\vec{\alpha}] |EPR\rangle = \frac{\hbar}{2} \alpha_x (\hat{G}_L[\vec{e}] + \hat{G}_P[\vec{e}]) \frac{1}{\sqrt{2}} (|\ell\vec{e}\rangle |\nu\vec{e}\rangle - |\ell\vec{e}\rangle |\nu\vec{e}\rangle) = \\ \sim \frac{\hbar}{2} \alpha_x (|\ell\vec{e}\rangle |\nu\vec{e}\rangle - |\ell\vec{e}\rangle |\nu\vec{e}\rangle + |\ell\vec{e}\rangle |\nu\vec{e}\rangle - |\ell\vec{e}\rangle |\nu\vec{e}\rangle) = 0$$

$$2) \quad \vec{\alpha}_1 = \alpha_1 \vec{o} \quad \vec{o} \cdot \vec{e} = 0 \quad |\vec{o}| = 1 \\ \hat{G}[\vec{\alpha}_1] |\vec{e}\rangle = \exp(-i\gamma) \alpha_1 \hat{G}[\vec{o}] |\vec{e}\rangle = \exp(-i\gamma) \alpha_1 |\vec{e}\rangle \quad \Leftarrow \text{viz. očekáváme, že dvojice } \vec{o}, \vec{e} \text{ kolmá má } \vec{e} \cdot \vec{o} = 0$$

$$\begin{aligned} \hat{S}[\vec{\alpha}_1] |EPR\rangle &= \frac{1}{\sqrt{2}} \frac{1}{\alpha_1} \exp(i\gamma) (\hat{G}_L[\vec{\alpha}_1] + \hat{G}_P[\vec{\alpha}_1]) (|\ell\vec{e}\rangle \underbrace{\hat{G}_P[\vec{\alpha}_1] |\nu\vec{e}\rangle}_{\sim |\nu\vec{e}\rangle} - \underbrace{\hat{G}_L[\vec{\alpha}_1] |\ell\vec{e}\rangle}_{\sim |\ell\vec{e}\rangle} |\nu\vec{e}\rangle) \\ &= \frac{1}{\sqrt{2}} \frac{1}{\alpha_1} \exp(i\gamma) (\hat{G}_L[\vec{\alpha}_1] + \hat{G}_P[\vec{\alpha}_1]) (\hat{G}_P[\vec{\alpha}_1] - \hat{G}_L[\vec{\alpha}_1]) |\ell\vec{e}\rangle |\nu\vec{e}\rangle \\ &= \frac{1}{\sqrt{2}} \frac{1}{\alpha_1} \exp(i\gamma) (\hat{I} - \hat{I}) |\ell\vec{e}\rangle |\nu\vec{e}\rangle = 0 \end{aligned}$$

$$\Rightarrow \hat{S}[\vec{\alpha}_1] |EPR\rangle = 0 \quad \# \vec{o}$$